Minimizing Energy Consumption in Hexapod Robots

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Abstract
Minimization of energy expenditure in autonomous mobile robots for industrial and service applications is a topic of huge importance, as it is the best way of lengthening mission time without modifying the power supply. This paper presents a method to minimize the energy consumption of a hexapod robot on irregular terrain. An energy-consumption model is derived for statically stable gaits before applying minimization criteria. Then, some parameters that define the foot trajectories of the legged robot are computed to minimize energy expenditure during every half a locomotion cycle. The method is evaluated using an accurate geometric model of the SILO-6 walking robot.


Keywords
Energy efficiency, energy consumption, legged locomotion, mobile robots, optimization methods

1. Introduction
An autonomous mobile robot is an intelligent machine with the skills to perform motion in an unstructured environment without explicit human intervention. Its intelligent autonomy is provided by efficient algorithms capable of coping with many different situations. Nevertheless, a real-application autonomous robot must also exhibit energy autonomy; the greater the autonomy, the better. Both sorts of autonomies are required for real operative mobile robots. Intelligent autonomy is provided by sensors and algorithms. Energy autonomy is mainly accomplished using efficient power supplies. However, the control algorithms that mobile robots use can help lengthen the duty cycle of power supplies and so extend the robot’s operation time. This is especially significant in legged locomotion (both multi-legged and biped robots), which can move along a given path with many different leg and body poses and, thus, with many different energy expenditure rates. Even though multi-
legged and biped robots exhibit many different features, they can share common energy optimization criteria [1], as presented below.

Different approaches to the energy efficiency of multi-legged robots have been studied. Some results have been obtained from the gait-parameter standpoint alone [2]. However, such models are extremely simplified, and do not consider the forces and torques involved in the process, which must be factored into energy computations. Early work for minimizing power consumption considered the actuator dynamics [3]; however, the theory was applied to hydraulic actuators, while we are interested in DC motors. Afterwards, Nahon and Angeles [4] worked on minimizing the power of robotic systems with DC motors, but allowed power regeneration by motors performing negative work; this capability is not offered by current technology. Later, Marhefka and Orin studied the force distribution problem in walking robots that minimizes the power consumption in DC motors disabling regeneration of negative work [5]. These results were then used to minimize the total power along a locomotion cycle rather than at one given instant [6]. There are other authors that have taken into account leg dynamics and torques [7–9] but they have still failed to consider joint actuator type, although the joint actuator’s contribution to energy consumption is decisive. Additionally, they focused their works on walking locomotion over flat terrain.

Normally, a walking robot is commanded to follow a set direction of motion at a set speed and then the control algorithms try to follow that direction at the given speed while negotiating any terrain irregularities. If the terrain is known in advance, the control algorithm can select the most adequate footholds to minimize energy expenditure during a locomotion cycle. Hence, this paper focuses on minimizing the energy expended by hexapod walking robots endowed with a perception system capable of generating elevation maps of the terrain ahead of them. The most important and new feature of the algorithm presented in this paper is its applicability to irregular terrain, while addressing DC actuators without regeneration of negative work.

Some examples of robots using systems to generate terrain-elevation maps can be found in the AMBLER and Dante II projects, which used laser rangefinders to map terrain elevation [10, 11]. The AMBLER system used spinning and nodding mirrors to scan its laser beam over the terrain to produce range images. The range images helped to measure the distance between the robot and visible points on the terrain. The Nomad project, devoted to developing robotic technologies for planetary exploration, is another example of a system that uses elevation maps. The Nomad system takes two stereo-camera images that are processed into range data. These range data are projected onto an overhead view, giving an elevation map of the area in front of the robot [12]. Recently, some new algorithms have been proposed based on a two-dimensional laser rangefinder to generate terrain-elevation maps [13]. Another algorithm for wheeled robots scans the terrain into local elevation maps and combines these local elevation maps by classifying the cells during data association [14].
Thus, we can see the importance elevation maps have held for autonomous mobile robots on irregular terrain for about the last decade. Elevation maps have been used mainly for navigation purposes in general, and, in some legged-robot applications, elevation maps have also provided a starting point for finding adequate footholds to improve traction and reduce foot slippage. However, terrain information can also help reduce the support torques of legged robots and, thus, reduce the energy required.

This paper focuses on the minimization of energy expenditure in hexapod robots walking on irregular terrain based on a priori knowledge of the terrain in front of the robot. This work is motivated by a real need to increase the energy autonomy of the SILO-6 walking robot. The SILO-6 is a six-legged robot developed as a mobile platform for the DYLEMA project [15, 16], whose main aim is to develop a locomotion system to integrate relevant technologies in the fields of legged robots and sensors to address needs in humanitarian demining activities. The hexapod is endowed with a 5-d.o.f. manipulator in front, which handles a sensor head to scan the terrain in the front of the hexapod covering its whole sprawl (see Fig. 1). This scanning manipulator is fast enough to scan the terrain without jeopardizing the hexapod speed. The sensor head consists of a mine-detecting set and a network of infrared (IR) sensors. The detecting set is a commercial device capable of detecting very small metallic components. The IR sensors are used to maintain the sensor head at a given height above the terrain and roughly parallel to any terrain irregularities. Thus, this sensor head can be used to generate an elevation map of what lies in front of the robot, which can in turn be used to minimize the energy required to move the robot. Figure 2 shows an example of reconstruction of an irregular terrain composed of different geometric objects using the described sensor head and a
Figure 2. Example of terrain elevation map obtained with the sensor head. (a) Real model that consists of prisms and cylinders on a flat terrain. (b) Reconstruction of the terrain under the sensor head.

tailor-made linear manipulator for preliminary tests. Note that the map width must be as wide as the hexapod sprawl.

This paper is organized as follows. Section 2 introduces the working hypotheses. Section 3 presents the energy model of a DC motor and extends the concept to find an energy model for a legged robot. Section 4 states the mechanical model of the robot required to compute the energy model. Section 5 introduces the optimization algorithm designed to minimize energy consumption and presents some simulation results. Section 6 states the practical implementation issues. Finally, Section 7 summarizes the main results and presents some conclusions.

2. Problem Statement

The main idea is to optimize the energy consumption of a six-legged robot walking on irregular terrain. Such a robot can perform a variety of gaits featuring very different properties. It is well known that a hexapod performing an alternating-tripod gait can achieve its highest speed [17] and, due to the fact that statically stable walking robots are intrinsically slow, alternating tripods becomes the most used gait by both natural and artificial hexapods. Thus, we focus our study on an alternating-tripod gait.

For optimization purposes, we consider the following conditions:

C-1 The center of gravity (COG) of the body follows a straight horizontal line with the body leveled at a given height with respect to a fixed reference frame. That means the body does not dip up and down as it moves horizontally. Hence, the robot does not expend any energy to raise the body that is not recovered when the body drops.
C-2 The robot’s body moves at a constant speed and does not have to withstand any external forces except its weight, i.e., no energy consumption is required to accelerate and decelerate the body. Nevertheless, inertial effects caused by the motion of the legs are considered in this study.

C-3 The robot performs an alternating-tripod gait, which means the robot is supported on three legs with fixed footholds for every half-cycle. Therefore, by optimizing the energy expended in each half-cycle, we optimize energy along an entire locomotion cycle and, thus, along any possible trajectory.

C-4 The alternating tripod is based on a fixed sequence of motion and the leg stroke, \( R_x \), is constant.

C-5 We assume the robot walks on practicable irregular terrain. Therefore, there are no forbidden zones, and every foot can be placed on the ground and provide support to the robot.

C-6 The robot’s actuators are based on DC electric motors. Thus, any negative energy (gain in energy supplied by external forces) is lost, because electrical motors cannot store such energy gains [18, 19].

3. Energy Consumption in a Walking Robot

The energy consumption in a robot is given by the sum of the energy consumed in every joint plus the energy consumed by electronic equipment (computers, drivers, analog I/O, etc.). The contribution of this last term is out of the scope of this work.

If a joint is driven by an electrical motor, the consumed energy is given by:

\[
E(t) = \int u_{\text{app}}(t)i(t)\,dt, \tag{1}
\]

where \( u_{\text{app}}(t) \) is the applied voltage and \( i(t) \) is the current through the motor.

Figure 3 shows the well-known electrical model of a motor where [19]:

\[
\begin{align*}
  u_{\text{app}}(t) - u_{\text{bef}}(t) &= Ri(t) + L\frac{d}{dt}i(t) \tag{2} \\
  u_{\text{bef}}(t) &= k_E\omega(t) \tag{3} \\
  \tau(t) &= k_Mi(t), \tag{4}
\end{align*}
\]

where \( R \) is the electric resistance of the motor, \( k_E \) is the back electromotive constant, \( k_M \) is the torque constant and \( \tau(t) \) is the motor torque. As a first approximation to the model, we consider the rotor inductance, \( L \), to be null. This is a normal practice in modeling DC motors for robotic joints [19, 20]. Note that in SI units the value of \( k_M \) is the value of \( k_E \), i.e., \( k_M \) (Nm/A) = \( k_E \) (Vs).

By substituting in (1), we obtain:

\[
E(t) = \int u_{\text{app}}(t)i(t)\,dt = \int \left( \omega(t)\tau(t) + \frac{R}{k_M^2}\tau^2(t) \right)\,dt, \tag{5}
\]
where the term $\omega(t)\tau(t)$ is the mechanical energy and the term $(R/k^2_{M_j})\tau_j^2(t)$ is the energy loss by heat emission. This second term is the energy consumed when the robot stands still, i.e., when the motors just support the robot with no motion at all.

Under condition C-6 (see Section 2), the negative mechanical energy is lost, because an electrical motor cannot store energy; therefore, the energy, $E_{ij}$, consumed by the motor of joint $j$ of leg $i$ during a time $T$ is finally given by:

$$E_{ij}(t) = \int_0^T \left( \Delta(\omega_{ij}(t)\tau_{ij}(t)) + \frac{R_j}{k^2_{M_j}}\tau_{ij}^2(t) \right) dt,$$

where the function $\Delta(x)$ is defined as:

$$\Delta(x) = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x \leq 0. \end{cases}$$

(7)

Taking into consideration the contribution of all joints in the hexapod, the total energy becomes:

$$E = \int_0^T \sum_{i=1}^6 \sum_{j=1}^3 \left( \Delta(\omega_{ij}(t)\tau_{ij}(t)) + \frac{R_j}{k^2_{M_j}}\tau_{ij}^2(t) \right) dt.$$

(8)

The main goal of this work is to minimize the expended energy $E$ given by (8). That energy depends on the torque and speed of every joint, which depend on physical parameters, such as the robot’s weight and body speed, and gait parameters, such as foot trajectories in the body reference frame, body height, etc. We assume the robot’s weight, $W$, does not change and the body speed is constant (condition C-2); thus, energy consumption depends on the robot’s height and foot trajectories. If the robot’s height is imposed by the application or task, then energy depends only on the foot trajectories. Some previous works consider the specific resistance, $\varepsilon$, for optimization [2, 6] or evaluation [1] purposes. This vehicle feature is defined as:

$$\varepsilon = \frac{E}{WL} = \frac{P}{Wv},$$

(9)

where $E$ is the energy required to travel a distance, $L$, by a vehicle with weight $W$. $P$ is the power consumed and $v$ is the speed of the vehicle. The specific resistance...
is useful to compare the efficiency of different types of vehicles. It is a kind of relative comparison that considers the power consumed per unit of mass and unit of speed. In our case, the energy expenditure depends on the algorithms rather than on the vehicle features. Weight and speed do not change (condition C-2); therefore, minimum energy also means minimum specific resistance and, for the sake of simplicity, this optimization process is focused on the energy.

With those premises, the only magnitude we can vary to optimize the energy is the distance of the footholds to the origin of the leg reference frame, because the leg stroke, $R_x$, is constant (condition C-4). Figure 4 illustrates the related parameters.

4. Computing the Energy

4.1. Basic Algorithm

Section 3 finds the energy consumed in moving and supporting a legged robot, which depends on the torque and speed of each motor. Those magnitudes depend on the torques and speed in joints, which can be computed from the mechanical and geometric parameters of the robot. Taking into consideration conditions C-1 and C-2, the robot can be studied as a quasi-static process because it moves at a constant speed (no acceleration) and external forces are neglected. In such a case, the joint torques of leg $i$ can be computed with the Jacobian matrix, $J_i$, as:

$$\tau^*_i = J_i^T f_i,$$

where:

$$\tau^*_i = (\tau^*_i 1 \tau^*_i 2 \tau^*_i 3)^T,$$

is the torque vector of leg $i$, 1, 2 and 3 define the joint in leg $i$, and:

$$f_i = (f_{ix} f_{iy} f_{iz})^T,$$

is the ground reaction force on foot $i$ in the body reference frame $(x, y, z)$ located in the geometric center of the robot’s body (see Fig. 4). Note that in (11) $\tau^*_i j$ denotes the torque in joint $j$ of leg $i$, while in:

$$\tau_i = (\tau_{i1} \tau_{i2} \tau_{i3})^T$$
\( \tau_{ij} \) denotes the torque in motor \( j \) of leg \( i \), which is defined by (4).

The computation of foot forces is termed ‘the force-distribution problem’. Let us assume that:

\[
\mathbf{F}_{pqr} = (\mathbf{f}_p \ \mathbf{f}_q \ \mathbf{f}_r)^T
\]

is the foot-force vector in the body reference frame when legs \( p, q \) and \( r \) are in support, where \( \mathbf{f}_i \) is the ground-reaction force in foot \( i \) defined by (12), and that the wrench \( \mathbf{W}_{pqr} \) contains the forces \( (F_x \ F_y \ F_z)^T \) and moments \( (M_x \ M_y \ M_z)^T \) acting on the robot’s COG, and represents the robot’s payload, any external applied load and the inertia effects of the robot’s body, i.e.:

\[
\mathbf{W}_{pqr} = (F_x \ F_y \ F_z \ M_x \ M_y \ M_z)^T. \tag{15}
\]

Under this condition, the equilibrium equations that balance forces and moments when three legs \( (p, q \) and \( r) \) are in their support phase are normally written in matrix form as [21, 22]:

\[
\mathbf{A}_{pqr} \mathbf{F}_{pqr} = -\mathbf{B}_R \mathbf{W}_{pqr}, \tag{16}
\]

where:

\[
\mathbf{A}_{pqr} = \begin{pmatrix}
\mathbf{I}_3 & \mathbf{I}_3 & \mathbf{I}_3 \\
\mathbf{R}_p & \mathbf{R}_q & \mathbf{R}_r \\
\end{pmatrix}, \quad \mathbf{B}_R = \begin{pmatrix}
\mathbf{I}_3 & \mathbf{0}_3 \\
\mathbf{R}_R & \mathbf{I}_3 \\
\end{pmatrix}. \tag{17}
\]

\( \mathbf{I}_3 \) is the \((3 \times 3)\) identity matrix, and \( \mathbf{R}_i, i \in \{p, q, r\} \), is the \((3 \times 3)\) skew symmetric matrix of the vector \((x_i, y_i, z_i)^T\) defined by the foot components of foot \( i \). \( \mathbf{R}_R \) is the skew symmetric matrix of the COG of the robot, \( \text{COG}_R \), i.e., the skew symmetric matrix of the vector \((x_{\text{COG}_R} \ y_{\text{COG}_R} \ z_{\text{COG}_R})^T\).

The pseudo-inverse method computes the foot forces of (16) as:

\[
\mathbf{F}_{pqr} = -\mathbf{A}_{pqr}^+ \mathbf{B}_R \mathbf{W}_{pqr}, \tag{18}
\]

where matrix \( \mathbf{A}_{pqr}^+ \) is the pseudo-inverse of \( \mathbf{A}_{pqr} \) and is given by:

\[
\mathbf{A}_{pqr}^+ = \mathbf{A}_{pqr}^T (\mathbf{A}_{pqr} \mathbf{A}_{pqr}^T)^{-1}. \tag{19}
\]

Let us assume that:

\[
\mathbf{W}_{I_{pqr}} = (F_{Ix} \ F_{Iy} \ F_{Iz} \ M_{Ix} \ M_{Iy} \ M_{Iz})^T, \tag{20}
\]

is the vector referred to the body reference frame \((x, y, z)\) formed by the inertial forces and moments acting in the COG of the body. These components are caused by the motion of the legs along a locomotion cycle. Then, the solution of (18) at instant \( k \) for the \( pqr \)-in-support locomotion cycle of the alternating-tripod gait is:

\[
\begin{pmatrix}
\mathbf{f}_p(k) \\
\mathbf{f}_q(k) \\
\mathbf{f}_r(k)
\end{pmatrix} = -\mathbf{A}_{pqr}^+(k) \mathbf{B}_R(k) \begin{pmatrix}
F_{Ix} \\
F_{Iy} \\
F_{Iz} - \mathbf{W} \\
M_{Ix} \\
M_{Iy} \\
M_{Iz}
\end{pmatrix}. \tag{21}
\]
These foot forces depend on the foot positions that determine matrices $R_{j,i}, i \in [p, q, r]$, and $R_R$, and in this case they are defined by the alternating-tripod gait (see Refs [21, 23] for foothold computation). Note that the wrench $\mathbf{W}_{pqr}$ in (21) contains the inertial forces and moments, and the external forces as well. Inertial forces and moments are caused by the motion of the masses of the leg links. These terms are obtained by computing first the accelerations of the COG of all of the links, COGL, along a semi-cycle. These accelerations are determined by the accelerations of the feet that will be defined below. The only external force acting in the robot is the weight, $W$, taking into consideration the condition C-2.

The solution of (21) along with (10) provides the joint torques. The speed in every joint is given by:

$$\omega_i^* = J_i^{-1}v_i,$$

where $\omega_i^*$ and $v_i$ are given by:

$$\omega_i^* = (\omega_{i1}^*, \omega_{i2}^*, \omega_{i3}^*)^T = (\dot{\theta}_{i1}^*, \dot{\theta}_{i2}^*, \dot{\theta}_{i3}^*)^T$$

$$v_i = (\dot{x}_i, \dot{y}_i, \dot{z}_i)^T.$$

$\theta_{ij}$ is the position of joint $j$ of leg $i$ and $(x_i, y_i, z_i)^T$ is the position of foot $i$. Again, $\omega_{ij}^*$ represents the speed in joint $j$ of leg $i$, while $\omega_{ij}$ represents the speed in motor $j$ of leg $i$.

The speed and torques in joint $j$ and motor $j$ of leg $i$ are related through the joint-gear ratio, $N_j$, and the joint gear’s mechanical efficiency, $\eta_j$, by:

$$\omega_{ij}^* = \frac{\omega_{ij}}{N_j}$$

$$\tau_{ij}^* = \eta_j N_j \tau_{ij}.$$

Therefore, (8) can be re-written as:

$$E = \int_0^T \sum_i \sum_j \left( \Delta \left( \frac{\omega_{ij}^*(t)\tau_{ij}^*(t)}{\eta_j} \right) + \frac{R_j}{k_M^2 \eta_j^2 N_j^2} (\tau_{ij}^*)^2(t) \right) dt.$$

Thus, the algorithm to compute the energy consumed in a hexapod consists of:

**Step 1.** Set the body velocity, body height, leg stroke $R_x$ and foot trajectories.

**Step 2.** Compute foot positions $(x_i(k), y_i(k), z_i(k))^T$ for instant $k$ using the alternating-tripod gait [21, 23].

**Step 3.** Compute the trajectory, speed and acceleration of every COG.

**Step 4.** With both the COG accelerations and link masses compute the inertial forces and moments.

**Step 5.** Compute foot forces $f_i$ by solving the force-distribution problem with (21).

**Step 6.** Compute torques $\tau_{ij}^*$ using (10).
Step 7. Compute joint speed $\omega^*_i$ with (22).

Step 8. Compute the total energy during half a locomotion cycle with (25), where $T = T_{cycle}/2$.

4.2. Kinematic Model of the Hexapod

Section 1 mentioned that the main aim of this work is to configure an energy-efficient walking robot for humanitarian demining activities. This robot model is considered herein for simulation purposes. The mechanical and geometric parameters of SILO-6 [15, 16] are defined in Fig. 4 and summarized in Table 1. With those parameters, a wire model of the robot can be obtained as illustrated in Fig. 5. The asterisk represents the location of the COG of each link. The COG of the body, COGB, is assumed to be at the geometric center of the body. With this model, we compute the position of the overall COG of the robot, COGR, as well as the forces and moments acting on it, which depend on the motion of the feet.

The foot positions have been designed to minimize the accelerations and, thus, to reduce the dynamic effects. The foot trajectory for each foot in an external reference frame will consists of a straight trajectory for the foot in its support phase and a transfer trajectory composed of three subtrajectories: a circular trajectory to rise the foot, a straight trajectory to move the foot forward and a circular trajectory to rise.

Table 1.
Main SILO-6 features

<table>
<thead>
<tr>
<th>Body dimensions (m)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>length $L_B$</td>
<td>0.88</td>
</tr>
<tr>
<td>front/rear width $D$</td>
<td>0.38</td>
</tr>
<tr>
<td>middle width</td>
<td>0.63</td>
</tr>
<tr>
<td>height</td>
<td>0.26</td>
</tr>
<tr>
<td>stroke pitch $P_x$</td>
<td>0.365</td>
</tr>
<tr>
<td>mass (kg)</td>
<td>28.2</td>
</tr>
<tr>
<td>speed (m/s)</td>
<td>0.05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Leg link length (m)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>0.084</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.250</td>
</tr>
<tr>
<td>$a_3$</td>
<td>0.250</td>
</tr>
<tr>
<td>stroke ($R_x$) (m)</td>
<td>0.25</td>
</tr>
<tr>
<td>mass (kg)</td>
<td>4.3</td>
</tr>
<tr>
<td>foot speed (m/s)</td>
<td></td>
</tr>
<tr>
<td>transfer phase</td>
<td>0.140</td>
</tr>
<tr>
<td>support phase</td>
<td>0.05</td>
</tr>
</tbody>
</table>

| Robot total mass (kg) | 54 |

See Fig. 4 for parameter definitions.
place the foot on the ground (Fig. 6b). With these trajectories, we guarantee that foot accelerations at foot takeoff and landing are minimized. Figure 6 plots the foot trajectory in both the body reference frame (Fig. 6a) and the external reference frame (Fig. 6b). Note that the foot trajectory in the body reference frame is transformed to joint trajectories by using the leg kinematic model described in the Appendix. Applying these joint trajectories into the hexapod’s controller (see a superposition of robot poses in Fig. 7), the inertial forces and moments acting in the COGR are computed.

It is worth clarifying that of the energy expended in the walking mechanism (stand, traction, leg swing and head loss), the contribution of the leg swing is insignificant. Every leg in its stand phase must support about one-third of the robot’s weight, while a leg in the swing phase does not support any load. That means the legs in the swing phase do not expend energy in comparison to the legs in the stand phase. Moreover, all the legs in the swing phase expend quite the same energy; therefore, the difference between two foothold solutions is basically the difference between their support phases. From the theoretical standpoint, there is, of course, a slight difference in the energy consumed for placing a foot in different points at the end of the swing phase, but they are quite close to each other and a swing phase is an unload motion, so this energy can be neglected. Additionally, the algorithm relies on an optimization method and the model must be kept significant enough, but yet small to be computationally efficient. This is the reason that in our model we discard the energy consumed in the swing legs.
Figure 6. Foot trajectories seen by an observer at (a) the body reference frame (trajectory in support from A to B) and (b) the external reference frame (C and D are the support points).

Figure 7. Superposition of robot poses showing the foot and COG_L trajectories.
4.3. Simulation Results

Let us compute the energy expended by the SILO-6 model when the robot walks along a straight, horizontal line with its body leveled and performing the foot trajectory plotted in Fig. 6 (see also Fig. 7). The body speed is assumed to be 0.1 m/s, which means the feet in support move at 0.1 m/s; however, the feet in the transfer phase reach up to 0.55 m/s; the leg stroke of the gait, $R_x$, is 0.25 m and the distance from every foot trajectory to the $z$-axis of its leg reference frame, $L_i$, is 0.25 m for feet in the support phase (tripod $i \in [1, 4, 5]$ in the half locomotion cycle under study). With these premises and the mechanical, geometric and electrical parameters provided in Tables 1 and 2, we obtain the motor torques and motor speed during half a locomotion cycle plotted in Fig. 8. These magnitudes are the base to obtain the power. For the sake of brevity, only the torques and speed for leg 1 are illustrated. Note that for an ideal mass-less leg robot the torque in motor 1 should

Table 2. SILO-6 joint parameters

<table>
<thead>
<tr>
<th>Leg joint</th>
<th>Motor</th>
<th>Torque constant $K_M$ (Nm/A)</th>
<th>Resistance $R$ (Ω)</th>
<th>Gear Rate $N$</th>
<th>Efficiency $\eta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0333</td>
<td>0.62</td>
<td>246</td>
<td>63</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.0333</td>
<td>0.62</td>
<td>881.5</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0333</td>
<td>0.62</td>
<td>881.5</td>
<td>42</td>
<td></td>
</tr>
</tbody>
</table>

Figure 8. Torques and speed in motors of leg 1 during half a locomotion cycle.
be null, because motor 1 does not contribute to robot support or motion at constant speed; however, Fig. 8 shows that in that joint the torque is not completely null because of the effect of the COG_L of the links. Note that in this case the robot is on flat, horizontal terrain.

Figure 9 plots the mechanical power consumption, along half a locomotion cycle, while legs 1, 4 and 5 are in support. Figure 10 plots the heat loss for the same legs and same conditions. These power contributions have been plotted individually for the sake of comparison. We can observe that heat loss is greater than mechanical power and the area under the heat loss (energy) is also greater than the area under the mechanical power. This fact highlights the importance of heat loss over mechanical power — a term that has been neglected in many studies. For instance, the pioneering work of Lapshin [2] considers only the energy consumption on the machine weight, the energy consumption on the traction generation and the energy for the leg swing phases. A more recent work [24] minimizes the sum of the square acceleration through the entire trajectory. However, by adding the two power contributions and integrating this function over half a locomotion cycle, we find that the mechanical energy required to support and move our hexapod is $E_M = 17.22$ J,
while the energy due to heat loss is $E_H = 37.01$ J. That makes a total energy of $E_T = 54.23$ J for a tripod (half a locomotion cycle), which means that the heat loss is about 68.24% of the total energy.

5. Minimization of Energy Consumption

Energy consumption depends on motor torque and motor speed, and these parameters depend on foot positions, as mentioned above. Therefore, varying the foot trajectories is one way of changing the energy required to support and propel a robot.

In our approach, the leg stroke, $R_x$, and leg phases (i.e., the leg-lifting instants in a locomotion cycle) remain constant. The period of the locomotion cycle will depend on the body speed. Foot trajectories in support are straight segments of a fixed length ($R_x$); thus, the only parameter for defining different foot trajectories is the distance from the foot trajectory to the $z$-axis of the leg reference frame, $L_i$, and the foot trajectory height, $H_i$ (see Fig. 4). Note that the robot is regarded as moving in a horizontal straight line with respect to a fixed external reference frame.
Therefore, if the robot walks on flat terrain, the \( H_i \) parameter becomes constant. If the robot walks on irregular terrain, \( H_i \) will depend on the \( x \) and \( y \) foothold components, and the only manner of finding footholds in advance will be to use sensors capable of providing the height, \( H_i \), as a function of the \( x \) and \( y \) foothold components:

\[
H_i = \begin{cases} 
H_{\text{Constant}} & \text{for flat terrain} \\
S(x_i, y_i) & \text{for irregular terrain.}
\end{cases}
\]  

(26)

For the application at issue here, obtaining the features of an irregular terrain is not a strong requirement, because, as mentioned in Section 1 the system (Fig. 1) can obtain terrain elevation maps in advance (Table 1).

To summarize the preceding paragraphs, we may say that the total energy, \( E \), expended during half a locomotion cycle of an alternating-tripod gait (wherein foot positions do not change in a fixed reference frame) is a function of the \( L_i \) parameters (see Fig. 4 for parameter definitions):

\[
E = \mathcal{E}(L_p, L_q, L_r),
\]  

(27)

where \( p, q \) and \( r \) are the legs that form the tripod for every half-cycle.

Let us assume the robot is walking on flat terrain at a height of \( H = -0.25 \) m; the foot trajectories are at \( L_p = L_q = L_r = L \), and we perform different experiments for different values of \( L \). Figure 11 plots the energy consumed by the robot during half a locomotion cycle as a function of \( L \). We can see there is a minimum for this special case where the terrain is flat and all \( L_i \) parameters are equal. Therefore, it is advantageous from the energy-consumption standpoint to compute the foot trajectories that minimized the energy.

5.1. Minimization Algorithm

Equation (27) is a scalar, nonlinear function of three variables, which can be minimized using the Nelder–Mead minimization algorithm: a multidimensional, un-
constrained, nonlinear minimization method also known as the flexible polyhedron method [25]. This method requires a starting point \( (L_p^0, L_q^0, L_r^0) \) and finds a local minimum \( (L_p^*, L_q^*, L_r^*) \) of the function \( \mathcal{E} \):

\[
\text{minimize}_{L_p, L_q, L_r} \{ \mathcal{E}(L_p, L_q, L_r) \} \rightarrow (L_p^*, L_q^*, L_r^*). \tag{28}
\]

Equation (25) is a unimodal function for the range of input values \( L_i \in [-0.1 \text{ m}, -0.35 \text{ m}] \) as numerically shown below (Fig. 11); therefore, the minimum found by (28) can be considered a global minimum. Note that the initial values of \( L_i \) are chosen to be inside the leg workspace so that the probability of finding a solution out of the leg workspace is negligible. However, if the inverse-kinematic function of the leg finds a point out of the workspace, it is not considered and the robot will perform the next step under non-optimal conditions.

5.2. Computing Results

To illustrate the algorithm results, let us consider that SILO-6 is walking on flat terrain at a height of \( H = -0.15 \text{ m} \), and apply (28) to compute the values of \( L_p \), \( L_q \) and \( L_r \) that minimize the energy. For the SILO-6 parameters summarized in Tables 1 and 2, and starting at the initial solution \( L_p^0 = L_q^0 = L_r^0 = 0.25 \text{ m} \), the optimal solution is \( L_p = 0.255 \text{ m}, L_q = 0.254 \text{ m} \) and \( L_r = 0.255 \text{ m} \), and the minimum energy is 49.63 J.

Let us now assume that the robot is walking on an irregular terrain that consists of a horizontal base plane with several protrusions, \( P \), such that the first tripod has footholds on the protrusions at \( H_i = -0.15 \text{ m} \) and the second tripod has footholds on the base plane at \( H_i = -0.30 \text{ m} \) (Fig. 12). If the robot walks during the first semi-cycle with foot trajectories at \( L_p = 0.255 \text{ m}, L_q = 0.254 \text{ m} \) and \( L_r = 0.255 \text{ m} \), the robot consumes the minimum energy as shown above. However, if the robot keeps the same foot-trajectory parameters for the second tripod, the energy expenditure is 52.04 J, whereas reapplication of the minimization algorithm at this point would reduce the energy to 50.42 J for \( L_p = 0.210 \text{ m}, L_q = 0.224 \text{ m} \) and \( L_r = 0.210 \text{ m} \). Note that the savings of 1.62 J is for a body motion of about \( R_x = 0.25 \text{ m} \) at a body

Figure 12. Sketch of the hexapod over irregular terrain.
Table 3.
Energy expenditure for different foot positions

<table>
<thead>
<tr>
<th>$H_i$ (m)</th>
<th>$-0.10$</th>
<th>$-0.15$</th>
<th>$-0.20$</th>
<th>$-0.25$</th>
<th>$-0.30$</th>
<th>$-0.35$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1^*$ (m)</td>
<td>0.248</td>
<td>0.255</td>
<td>0.250</td>
<td>0.235</td>
<td>0.210</td>
<td>0.074</td>
</tr>
<tr>
<td>$L_4^*$ (m)</td>
<td>0.244</td>
<td>0.254</td>
<td>0.252</td>
<td>0.242</td>
<td>0.224</td>
<td>0.197</td>
</tr>
<tr>
<td>$L_5^*$ (m)</td>
<td>0.248</td>
<td>0.255</td>
<td>0.250</td>
<td>0.235</td>
<td>0.210</td>
<td>0.074</td>
</tr>
<tr>
<td>Energy (J)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_i'$ (m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.10$</td>
<td>39.66</td>
<td>39.88</td>
<td>39.76</td>
<td>39.94</td>
<td>42.83</td>
<td>74.60</td>
</tr>
<tr>
<td>$-0.15$</td>
<td>49.82</td>
<td>49.63</td>
<td>49.66</td>
<td>50.35</td>
<td>53.46</td>
<td>70.60</td>
</tr>
<tr>
<td>$-0.20$</td>
<td>54.27</td>
<td>54.23</td>
<td>54.21</td>
<td>54.54</td>
<td>56.50</td>
<td>64.95</td>
</tr>
<tr>
<td>$-0.25$</td>
<td>54.36</td>
<td>54.65</td>
<td>54.49</td>
<td>54.23</td>
<td>54.88</td>
<td>58.13</td>
</tr>
<tr>
<td>$-0.30$</td>
<td>51.40</td>
<td>52.04</td>
<td>51.75</td>
<td>50.91</td>
<td>50.42</td>
<td>50.42</td>
</tr>
<tr>
<td>$-0.35$</td>
<td>46.19</td>
<td>47.12</td>
<td>46.73</td>
<td>45.42</td>
<td>44.04</td>
<td>41.60</td>
</tr>
</tbody>
</table>

speed of $0.1$ m/s. These values are shown in Table 3, which includes the energy expenditure under different body height, $H_i$, and leg spread, $L_i$. In Table 3, $L_i^*$ is the foot position that minimizes the energy (in bold) for a given $H_i$. $H_i'$ represents the height of the body at the computation of the energy. The rows in the energy part of Table 3 contain the energy expenditure for a given body height, $H_i'$, when the foot positions are those indicated in the same column. Thus, the minimum in every row appears at the cell $H_i = H_i'$ (indicated in bold). Note that for both support tripods the body is on the same horizontal plane; therefore, there is not any energy expended for moving up or down the robot’s body.

Figure 13 illustrates the energy surface as a function of $H_i$ and $H_i'$. The points of minimum energy for a given $H_i'$ are indicated with an asterisk. The surface shows how the minimum energy expenditure is obtained for low or high body heights, which correspond with the insect or mammal robot configuration. However, for a given body height, $H_i'$, the minimum energy is obtained at a specific foot positions, $L_i$.

Similar examples can be run considering other type of irregular terrain. For instance, the first tripod could be at the base plane with $H_i = -0.15$ m and the second tripod could be inside holes such as $H_i = -0.30$ m. The results in both examples are the same. A general irregular terrain with the footholds at different heights can also be used, but we prefer to present a simple irregular terrain for the sake of clarity.

6. Practical Implementation Issues

The earlier sections have presented the theory for minimizing the energy consumption in a hexapod robot when it performs a tripod gait along a straight line.
A numerical example of the procedure has illustrated that it is possible to save about 3.21% of the energy for a hypothetical, special case where every foot in a tripod is at the same height. In such a case, a lookup table with the solution to the problem could be an adequate practical solution to avoid computing the optimization procedure in real-time.

In a practical case, when the robot is walking on irregular terrain every foot will normally be placed at a different height in a range from $-0.10$ m up to about $-0.35$ m. The lookup table solution seems to be unpractical, even if we use a reasonable discretization of the foot heights, and the implementation in real-time of the minimization algorithm is the only sensible solution, which brings about further technical computing problems.

Let us assume that we know, as in our real application introduced in Section 1, an elevation map of the terrain where the robot is stepping on. That means we know the $z$-foothold component as a function of the $x$ and $y$ foothold components. Thus, we can compute the minimization algorithm at the beginning of the transfer phase, which lasts for about $t_T = 2.5$ s, taking into consideration the average speed of the transfer phase $v_T = 0.1$ m/s and the leg stroke $R_x = 0.25$ m. This time $t_T$ is insufficient for running the minimization algorithm in the PC-bus based computer (industrial ISA PC Pentium II 667 MHz) on board SILO-6. Additional computing resources must be provided on the prototype to implement the method presented in this paper. This is part of the technical work envisaged for the next step of the

Figure 13. Energy expenditure for different body heights.
In this project. Nevertheless, the most important result is to quantify the range of energy consumption we can save by applying minimization methods.

7. Conclusions

Energy optimization is an important topic for autonomous robots, because it is the way of increasing duty time for missions without modifying the power supply on board the robot.

This paper studies the energy required for six-legged robots using alternating-tripod gaits, the fastest gait for hexapods, and derives a method to minimize the energy for one tripod. Minimization of consecutive tripods minimizes the energy throughout an entire trajectory. The method works on any kind of irregular terrain, although some simple models for irregular terrain have been used in the examples presented above.

The method requires some advance knowledge of the terrain; therefore, it is applicable to robots capable of obtaining an elevation map of the terrain. Energy expenditure was computed by using the SILO-6 walking robot as a model. This robot is designed for humanitarian demining missions and uses a system to obtain an elevation map of the terrain in front of the robot.

This paper provides some figures for the energy saved in such a robot and research is ongoing on the computation of the method for real-time applications. Energy consumed by electronics (computers, drivers, analog I/O, etc.) constitutes a large source of energy loss. Diminishing this energy loss is a technical concern that must always be taken into account.

Acknowledgements

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References


Appendix

This Appendix presents the leg kinematics used for simulation and control purposes (Fig. A.1). The direct kinematic relationships consist in envisioning the position \((x_i, y_i, z_i)^T\) of foot \(i\) as a function of the joint components \((\theta_{i1}, \theta_{i2}, \theta_{i3})^T\):

\[
(x_i, y_i, z_i)^T = T_{\text{DIR}}(\theta_{i1}, \theta_{i2}, \theta_{i3})^T,
\] (A.1)
where

\[
T_{\text{DIR}} = \begin{pmatrix}
C_{\theta_11}(a_3C(\theta_12 + \theta_13) + a_2C\theta_12 + a_1) \\
S_{\theta_11}(a_3C(\theta_12 + \theta_13) + a_2C\theta_12 + a_1) \\
a_3S(\theta_12 + \theta_13) + a_2S\theta_12
\end{pmatrix}.
\] (A.2)

Note that we simplify the notation by writing C for cos and S for sin.

The inverse kinematics is defined by:

\[
(\theta_{i1} \ \ \theta_{i2} \ \ \theta_{i3})^T = T_{\text{INV}}(x_i, y_i, z_i) \quad \text{for } i = p, q, r,
\] (A.3)

where:

\[
T_{\text{INV}} = \begin{pmatrix}
\arctg \left( \frac{y_i}{x_i} \right) \\
- \arctg \left( \frac{B_i}{A_i} \right) + \arctg \left( \frac{D_i}{\pm \sqrt{A_i^2 + B_i^2 - D_i^2}} \right) \\
\arctg \left( \frac{z_i - a_2S\theta_12}{x_iC\theta_11 + y_iS\theta_11 - a_2C\theta_12 - a_1} \right) - \theta_{i2}
\end{pmatrix},
\] (A.4)

and:

\[
A_i = -z_i \\
B_i = a_1 - x_iC\theta_11 + y_iS\theta_11 \\
D_i = \frac{2a_1(x_iC\theta_11 + y_iS\theta_11) + a_2^2 - a_2^2 - a_1^2 - x_i^2 - y_i^2 - z_i^2}{2a_2}.
\] (A.5)
Finally, the Jacobian matrix is:

\[
\mathbf{J} = \begin{pmatrix}
-S\theta_1(a_3C(\theta_2 + \theta_3) + a_2C\theta_2 + a_1) \\
C\theta_1(a_3C(\theta_2 + \theta_3) + a_2C\theta_2 + a_1) \\
0 \\
-C\theta_1(a_3S(\theta_2 + \theta_3) + a_2S\theta_2) & -a_3C\theta_1S(\theta_2 + \theta_3) \\
-S\theta_1(a_3S(\theta_2 + \theta_3) + a_2S\theta_2) & -a_3S\theta_1S(\theta_2 + \theta_3) \\
-a_3C(\theta_2 + \theta_3) + a_2C\theta_2 & a_3C(\theta_2 + \theta_3)
\end{pmatrix}.
\] (A.6)

Joint reference frames and leg parameters are defined in Fig. A.1. Link length values, \(a_i\), are indicated in Table 1.

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