Robust Regression Applied to Ultrasound Location Systems

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Abstract **– Local positioning systems (LPS), specially those using ultrasound, are able to accurately estimate the location of persons or objects indoors. However, under certain circumstances, its accuracy can be strongly deteriorated by outlying noise. This paper analyzes and compares several strategies for robust trilateration, such as high-breakdown-point robust methodologies, as well as the parity space outlier detection procedure, which is commonly used in GPS. This analysis is performed by simulation in a typical ultrasound location system scenario based on the actual location of nodes in the 3D-LOCUS system [1]. It is shown how the traditional parity space outlier detection method overcomes robust methodologies when only one ranging error is present, whereas it is not able to detect two simultaneous faults. It is proposed a modification of the LTS robust estimation methodology that offers a good performance even when several range measurements are erroneous, due to multipath and occlusions effects. The complexity of the robust algorithms studied is low enough for being implemented in the 3D-LOCUS system without affecting its current 10 Hz update rate.**

I. Introduction

Nowadays, location technologies have started to be present in every day life. The most extended technology is the global positioning system NAVSTAR GPS. Accuracies achieved by GPS range from tens of meters to a few centimeters [2], depending mainly on the method used for measuring the time of arrival, the visibility of satellites, and the multipath present in the received signals. Visibility is restricted by the blockage of signals caused by walls, buildings or forest. For these areas where the GPS is not available there are alternative solutions called Local Positioning Systems (LPS) that can operate even indoors.

While GPS is a mature technology that provides location information worldwide, there is not any equivalent system for local positioning that has prevailed. The location accuracy of LPSs and the infrastructure to install depend on the technology used (e.g. radio, vision, ultrasound). Those based on radio signals require less infrastructure and achieve accuracies ranging from several meters [3][4] (Wifi and RFID LPSs) to tens of centimeters [5][6] (UWB LPS). Artificial vision systems provide accuracies from several centimeters [7] to several millimeters with a very expensive infrastructure, low modularity and high processing requirements. Systems based on ultrasound signals achieve a centimeter-level accuracy, being very flexible, with high modularity, and low processing demands.

Traditional ultrasound location systems (US-LPS), such us Active Bat [8] and Cricket [9], use highly directive resonant transducers, requiring an extensive infrastructure. Other new developments, such as Dolphin units [10] and 3D-LOCUS [1], try to emulate the Global Positioning System through the use of broadband transducers with wide lobe. The use of broadband transducers permits the systems to use codified signals, increasing the accuracy and enabling Code Division Multiple Access (CDMA) schemes. The use of transducers with wide emission and reception lobes makes them to require less infrastructure.

All these positioning systems are subject to de influence of erroneous measurements, which affect positioning accuracy and reliability. There is an extensive bibliography on GPS concerning positioning reliability and outlier detection. The former is achieved through RAIM [11] (Receiver Autonomous Integrity Monitoring) algorithms executed independently in every receiver. The latter employs FDE (Fault Detection and Exclusion) algorithms and reference stations [12] for detecting failing satellites. However, in US-LPS, there is not any important study which evaluates positioning reliability nor detection of erroneous measurements.

This paper analyzes different outlier resistant methodologies

TABLE I.
MAIN CAUSES OF COMMON RANGE ERRORS PRESENT IN GPS AND ULTRASOUND LOCATION SYSTEMS

System	Type of ranging error			
	Gaussian	Ramp	Peaks	Steps
GPS	SNR	Clock Drift	Clock error	Maintenance or Ephemeris change
US-LPS	SNR	Diffraction	Multipath	Strong multipath

aiming at the selection of the best algorithm for ultrasonic LPSs, such as the 3D-LOCUS system. It is important for the chosen algorithm to be robust under simultaneous ranging errors, preserving a high accuracy and with a computation time that do not influence the 3D-LOCUS update rate.

Next section will describe the causes of ranging errors that apply in GPS and ultrasound positioning. Section III shows the basic iterative algorithm employed in the simulations for ultrasonic trilateration. Next, the implementation of the parity space method as well as some robust algorithms are presented. In section V the evaluation of the algorithms is developed. And finally, a discussion and conclusions are presented, stressing practical implications of presented results and future research.

II. Ultrasound Versus GPS Ranging Noise

When studying GPS and Ultrasound Location scenarios, several important differences arise concerning erroneous measurements characteristics. In general, except for the gaussian noise, the measurements are affected by three types of errors, namely, peaks (intermittent faults), steps (systematic errors that are constant over time), and ramps (a linearly changing error in range) [13]. These errors have different causes depending on the system under study (see Table I).

In GPS, the ramp errors are mainly produced by clock drifts. Also, peak errors are due to clock errors [13]. These errors are due to the high precision and stability required for the clock, that highly influences range measurements. The step error can be produced by several causes in GPS: power interruption, maintenance, or change of ephemeris data. Multipath effect usually causes a correlation peak deformation [12] (the difference between the direct path signal and the multipath signal is usually considered below 300 m), causing an increment in the standard deviation of pseudorange measurements.

Meanwhile, in the field of the Ultrasound Location Systems, ramp errors appear when an object is placed in the line of sight (LOS) of the signal, partially occluding the direct path. Therefore, the signal, received by diffraction on the object's surface, is delayed by a certain amount which depends on the object's size and position. In the case of an object moving along the LOS, a ramp deviation in range measurements can be expected. With regard to the peak errors, they appear as a consequence of multipath, usually mixed with partial occlusion. In the presence of either a total occlusion or a multipath signal stronger than the LOS signal (strong multipath), the error measurement may become persistent for a period of time, which explains the typical step errors observed on corrupted measurements.

These three errors are very common in a real ultrasound scenario. Consequently, when studying the error influence, the simultaneous presence of more than one of such errors has to be taken into account.

III. Basic Trilateration Algorithm for Ultrasonic Positioning

The basic trilateration algorithm employed in 3D-LOCUS is based on absolute Time of Flight (ToF) measurements [1]. Temperature gradients, usually present in indoor environments, make it advisable to include the sound velocity estimation in the trilateration equations in order to obtain a more accurate position measurement. Trilateration equations are defined by:

$$t_i = \frac{\sqrt{(x_i-x_u)^2+(y_i-y_u)^2+(z_i-z_u)^2}}{V_s} \qquad (1)$$

where:

t_i is the measured ToF between the user and the fixed node "i",

(x_i, y_i, z_i) is the known position of the fixed node "i",

(x_u, y_u, z_u) is the unknown user position, and

V_s is the unknown sound velocity.

Every fixed node located in the environment defines an equation as Eq. 1. Linearizing these equations using a Taylor series around an approximate user's position and sound velocity ($\hat{x}_u$,$\hat{y}_u$,$\hat{z}_u$,$\widehat{V}_s$), we obtain:

$$\Delta t_i = \frac{x_i-\hat{x}_u}{\widehat{V}_s \hat{r}_i}\Delta x_u + \frac{y_i-\hat{y}_u}{\widehat{V}_s \hat{r}_i}\Delta y_u + \frac{z_i-\hat{z}_u}{\widehat{V}_s \hat{r}_i}\Delta z_u - \frac{\hat{r}_i}{\widehat{V}_s^2}\Delta V_s \qquad (2)$$

Or, rewritten in a compact matrix notation, considering the whole set of equations:

$$\mathbf{\Delta t} = \mathbf{H}\mathbf{\Delta x} \qquad (3)$$

where:

$$\mathbf{\Delta t} = \begin{bmatrix} t_1-\hat{t}_1 \\ t_2-\hat{t}_2 \\ \vdots \\ t_n-\hat{t}_n \end{bmatrix} \qquad (4)$$

$$\mathbf{H} = \begin{bmatrix} \frac{x_1-\hat{x}_u}{\widehat{V}_s\hat{r}_1} & \frac{y_1-\hat{y}_u}{\widehat{V}_s\hat{r}_1} & \frac{z_1-\hat{z}_u}{\widehat{V}_s\hat{r}_1} & -\frac{\hat{r}_1}{\widehat{V}_s^2} \\ \frac{x_2-\hat{x}_u}{\widehat{V}_s\hat{r}_2} & \frac{y_2-\hat{y}_u}{\widehat{V}_s\hat{r}_2} & \frac{z_2-\hat{z}_u}{\widehat{V}_s\hat{r}_2} & -\frac{\hat{r}_2}{\widehat{V}_s^2} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_n-\hat{x}_u}{\widehat{V}_s\hat{r}_n} & \frac{y_n-\hat{y}_u}{\widehat{V}_s\hat{r}_n} & \frac{z_n-\hat{z}_u}{\widehat{V}_s\hat{r}_n} & -\frac{\hat{r}_n}{\widehat{V}_s^2} \end{bmatrix} \qquad (5)$$

$$\mathbf{\Delta x} = \begin{bmatrix} \Delta x_u \\ \Delta y_u \\ \Delta z_u \\ \Delta V_s \end{bmatrix} \qquad (6)$$

with n fixed nodes.

Equation 3 has the solution

$$\mathbf{\Delta x} = \mathbf{H}^{+}\mathbf{\Delta t} \qquad (7)$$

where $\mathbf{H}^{+} = \left(\mathbf{H}^{T}\mathbf{H}\right)^{-1}\mathbf{H}^{T}$ is the Moore-Penrose pseudoinverse of matrix $\mathbf{H}$. Using this equation iteratively, the user position is obtained.

IV. Outlier Resistant Methodologies

Next, the implementation of RAIM techniques and parity space methods in the 3D-LOCUS system is presented. After that, the robust algorithms studied are explained: Least Median Squares (LMS) and Least Trimmed Squares (LTS). Finally, it is presented a slight modification to these robust methodologies, proposed by the author of this paper, that has demonstrated to improve localization results.

A. Parity Space Method

The most widespread method for integrity monitoring is the use of the parity vector [12], [11]. From equation 3, being the $\mathbf{H}$ matrix of dimensions $n \times 4$, it is defined the $\mathbf{P}$ matrix of dimensions $n-4 \times n$ such that its rank is $n-4$, $\mathbf{PP}^{T} = \mathbf{I}_{n-4}$, and $\mathbf{PH} = 0$. This matrix can be obtained through singular value decomposition [14] or QR factorization [15] of matrix $\mathbf{H}$; if $\mathbf{H} = \mathbf{QR}$ and $\mathbf{Q} = [\mathbf{Q}_1, \mathbf{Q}_2]$, then $\mathbf{P} = \mathbf{Q}_2^{T}$. The parity vector will be obtained as

$$\mathbf{p} = \mathbf{P\Delta t} \qquad (8)$$

The transformation of the parity vector to the measurement space is the faulty vector, which is obtained as

$$\mathbf{f} = \mathbf{S\Delta t} \qquad (9)$$

being $\mathbf{S} = \mathbf{P}^{T}\mathbf{P}$, with rank $n-4$ and idempotent ($\mathbf{S}^2 = \mathbf{S}$).

A common decision variable for detecting a faulty measurement is $D = \mathbf{p}^{T}\mathbf{p} = \mathbf{f}^{T}\mathbf{f}$. When it reaches a threshold it is deemed that there is an erroneous measurement. For identifying the faulty measurement it is calculated the maximum value of f_i^2/S_{ii}.

The detection threshold (T) is usually defined from the false alarm probability, defined as the probability of the decision variable being above the threshold without any measurement error. This probability will be defined as [11]:

$$P_{FA} = Q\left(\frac{T}{\sigma_n^2}\middle|\, n-4\right) = 1 - P\left(\frac{T}{\sigma_n^2}\middle|\, n-4\right) \qquad (10)$$

where $P(\chi^2|r)$ is the chi-square probability distribution with r degrees of freedom. Although this is the function commonly used for integrity evaluation, it is an approximation [16] that simplify the calculations.

B. LMS and LTS Methods

Due to the masking effect of the parity space method, the use of robust estimators for position calculation seems to be more appropriate. Several robust estimators have been tested in the simulations, being only presented two (LMS and LTS) as the most appropriate due to their high breakdown point [17]. These robust methods will take into account two or more simultaneous faults. Most robust algorithms (such as absolute value or Huber M-estimator) fail in the presence of large errors produced by multipath and impulsive noise.

Both algorithms involve the calculation of multiple solutions which are obtained applying the iterative algorithm presented in section III, to a subset of the measurement vector of length h ($h < n$) [18]. In this paper, the value of h is fixed to 5, and all the possible subsets of length h are considered, obtaining $\frac{n!}{h!(n-h)!}$ candidate solutions (consisting of the estimated values $\hat{x}_u$, $\hat{y}_u$, $\hat{z}_u$, and $\widehat{V}_s$). Then, the n residuals corresponding to each of these solutions are calculated ($\mathbf{\Delta t}_{1:n}$), subtracting the estimated ranges (using the equation 1 with the estimated values) from the measured ranges (t_i). Finally, the solution selected by the algorithms is based on this residuals:

Least Median of Squares (LMS) regression estimator consider that the best solution is the one that minimizes the median of the squared residuals, that is:

$$MED\left(\mathbf{\Delta t}_1^2, \mathbf{\Delta t}_2^2, \cdots, \mathbf{\Delta t}_n^2\right) \qquad (11)$$

Least Trimmed Squares (LTS) estimator will select the solution which minimizes

$$\textstyle\sum_{i=1}^{h} \mathbf{R}_i^2 \qquad (12)$$

where $\mathbf{R}_i^2$ are the n squared residuals ($\mathbf{\Delta t}^2$) written in ascending order.

These algorithms are able to overcome $n-h$ failing measurements. With h fixed to 5, up to two simultaneous faults can be overcome. It has to be taken into account, when selecting the value of h, that it must be chosen bigger than half the number of measurements and bigger than the number of parameters to be estimated.

C. Proposed Modification

Simulations presented on this paper will show that these robust algorithms (LMS and LTS) present large errors in several estimations. Observation of the estimated velocity of sound suggests the use of this variable for testing the goodness of the obtained result, since its value suffers a notorious variation when an error occurs in the estimation. The advantage of using this variable is that we possess an alternate method of estimating its value: temperature monitoring.

The mathematical dependence of the sound velocity on temperature is defined as:

$$V_s = 331.3\sqrt{1 + \frac{T}{273.15}} \qquad (13)$$

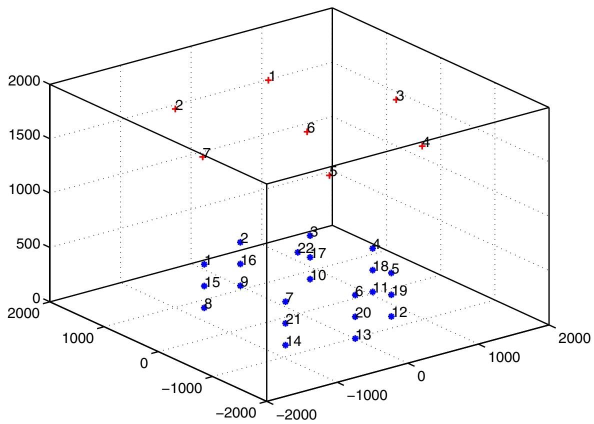


Fig. 1. BEACONS AND TEST POINTS DEFINITION

where T is the temperature measured in Celsius degrees.

The proposed method rejects the result obtained by LTS or LMS if its velocity of sound estimation differs by a certain amount of a measured value, selecting one of the remaining solutions applying again the corresponding minimization equation (eq. 11 for the LMS algorithm and eq. 12 for the LTS algorithm). This procedure is repeated until the velocity of sound estimation of the selected solution is close enough to the measured value. It does not change the speed of sound estimation obtained from ToF measurements, since the joint estimation of $\hat{x}_u$, $\hat{y}_u$, $\hat{z}_u$, and $\widehat{V}_s$, obtains better results.

V. SIMULATION RESULTS

The simulations accomplished take into account the actual spatial distribution of nodes in the 3D-LOCUS system [1]. The chosen configuration corresponds to the "private mode" of this system, where fixed nodes emit and the mobile node receives the ultrasound signals. Fig. 1 shows the fixed nodes emitters' location (red crosses) and the mobile node receiver positions on every test point (blue circles).

The 3D-LOCUS system consists of seven fixed nodes (also called beacons or satellites as an analogy to GPS) located in a fixed structure, and one mobile node attached to a robot arm as its tool (for precise positioning).

22 test points distributed in three different heights are contemplated. For every point, 100 independent measurements are simulated using as starting estimation the point located 1.5 m below the centroid of fixed nodes. Ideal ToF measurements are corrupted with gaussian noise with a standard deviation of 3 mm. V_s is fixed to 340 m/s (14.5°C).

The simulation takes into account four different error situations:

- Gaussian noise.
- Ramp error.
- Multipath error.
- Multipath and outliers errors.

Every error situation is repeated at each test position. The ramp error ranges from 0 to 100 mm in fixed node 1. The multipath persistent error is of 1 m and is applied to the fixed node 1. Outliers are present in a 10% of the measurements with a value of 1 m applied to the fixed node 3.

The results are not shown for the LMS algorithm for the sake of simplicity, since they are closely the same as the obtained with LTS. False alarm probability is fixed to 0.01. Sound Velocity checking algorithm will deem that an error is occurring when estimated velocity is not within the range 335-345 m/s (6-23 °C).

A. *Error Cumulative Distribution Function*

Fig. 2 shows the Cumulative Distribution Function of the error obtained for the four error situations with the three algorithms presented. It is shown that proposed algorithm has always approximately the same error distribution independent of the error situation. The method based on parity space outperforms robust methods in three out of four erroneous situations, whereas it is not able to detect the failing satellites when two large errors are considered.

Fig. 2(a) shows the results obtained when only gaussian error is added to the measurements. The best result is obtained with the linear method since it is the maximum likelihood algorithm for this case. LTS errors are higher than expected, due to the characteristic of the function evaluated, since it presents a global minimum with smaller residuals than other solutions closer to the real position. This is checked with the proposed algorithm, since it only discards some solutions present in the LTS algorithm, obtaining better results.

Similar results are obtained when a ramp error is present (Fig. 2(b)). In this case, parity space method is able to avoid the ramp error without being highly influenced. LTS error is highly influenced by this error which is repaired with the proposed method.

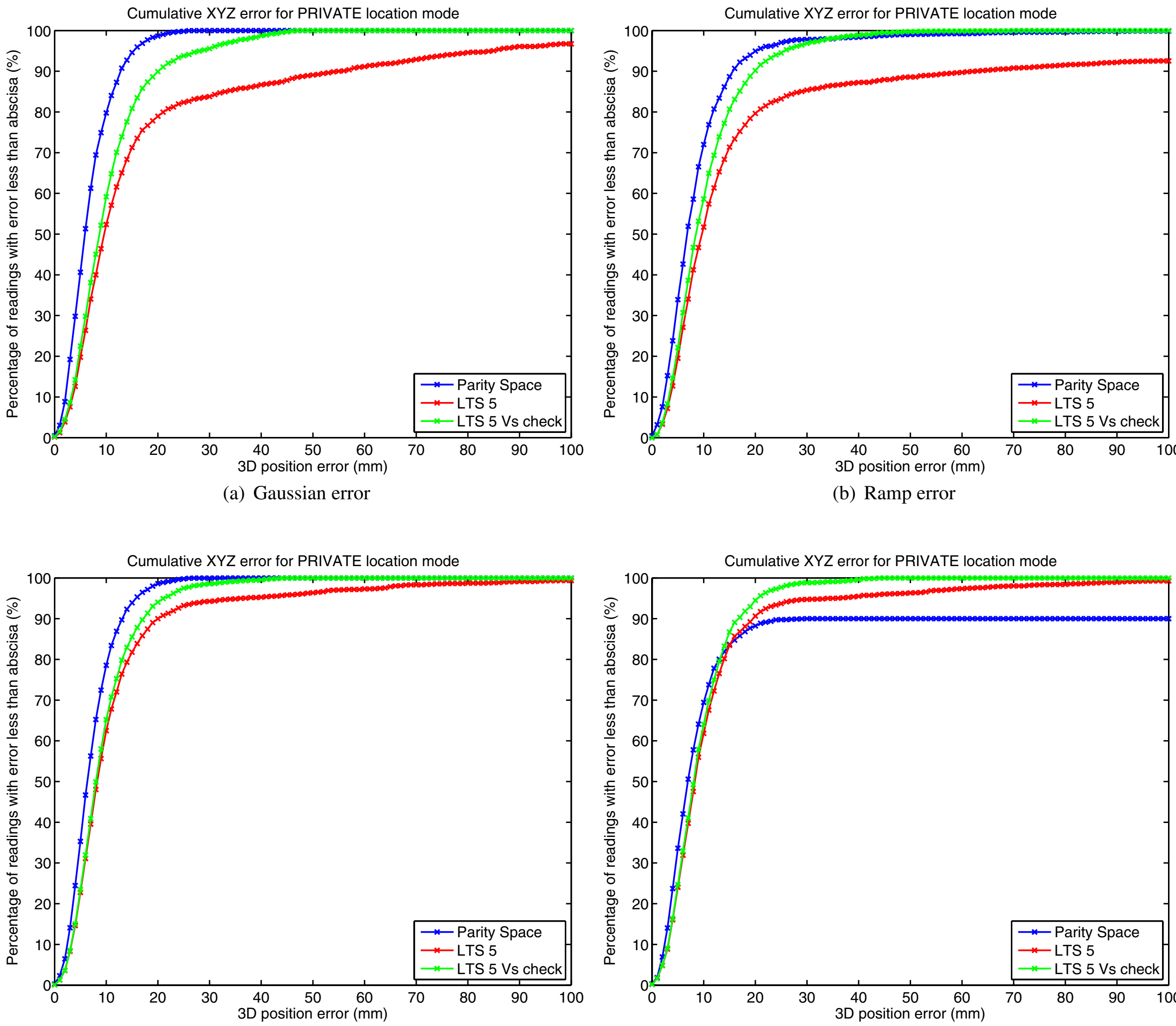
When a large error, due to multipath, is present (Fig. 2(c)), the LTS algorithm performance improves. Both other methods are not influenced by this error.

Finally, when the measurements are corrupted with two large errors, the parity algorithm fails (Fig. 2(d)). LTS and proposed algorithm performance is very similar to the previous case (the latter get better results). It is remarkable that LTS performance is better than the obtained with small errors.

B. *Test Position Failures*

After analyzing the global error performance, it is important to study the spatial distribution of large location errors. It has been taken as a large error those above 100 mm (those exceeding previous error distribution limits).

In the first three situations (gaussian noise, and ramp and multipath errors), there are no large errors when using neither the parity space method nor the proposed method. LTS algorithm shows an homogeneous distribution of the errors in



(a) Gaussian error

(b) Ramp error

(c) Multipath error

(d) Multipath and impulsive noise errors

Fig. 2. ERROR CUMULATIVE DISTRIBUTION FUNCTION OBTAINED FOR THE FOUR ERROR SITUATIONS SIMULATED

every test point in the first two (Fig. 3(a)). With multipath errors this algorithm has very few large errors.

In the last situation, two simultaneous errors are considered. Fig. 3(b) shows that the parity space algorithm fails to detect the erroneous measurements when both are present simultaneously. LTS method still presents an small number of large errors, that are overcome with the temperature monitoring methot.

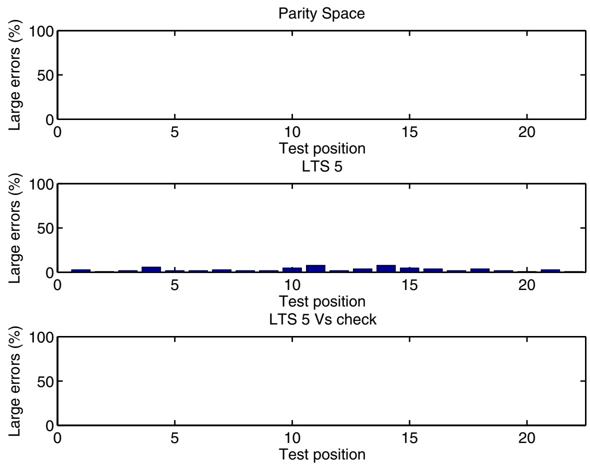
C. *Range measurements exclusion*

The main difference between parity space satellite exclusion and LTS methods is that the former excludes one measurement each time, sequentially, if any error is found, whereas the latter exclude always two at the same time. Fig. 4 shows the satellite exclusion performed by these algorithms. In LTS algorithms "first" represents the excluded satellite with lower number.

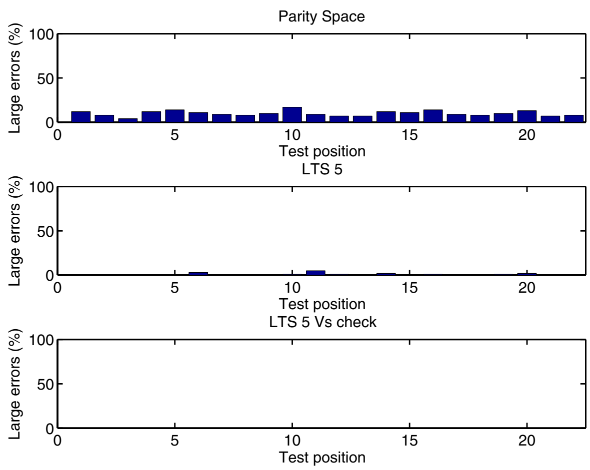
Fig. 4(a) shows the results obtained when a ramp error is introduced to fixed node 1. It is observed that every algorithm detect the failing measurement in a high percentage of trials. Vs checking increases this percentage.

When testing for the multipath error, Fig. 4(b) shows that every algorithm detects the failure present in satellite 1. Parity space detects that this is the only erroneous measurement.

Fig. 4(c) corresponds to the detected failures when testing with impulsive noise (10% of the measurements corrupted) and multipath. LTS algorithms always detect the multipath erroneous measurement (number 1). LTS basic algorithm fails to detect the second error in a small percentage, which is solved when checking the estimated sound velocity. Parity method

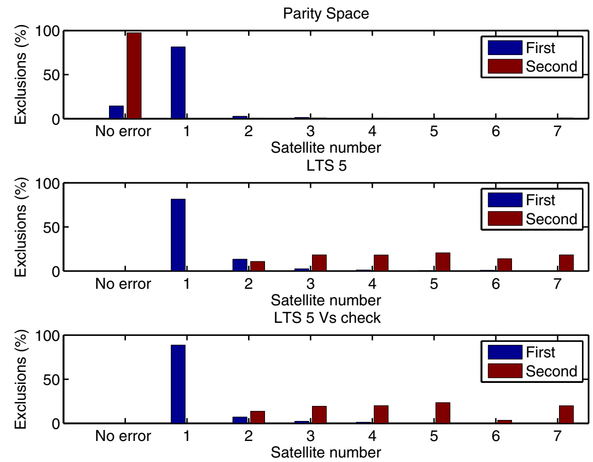


(a) Gaussian error

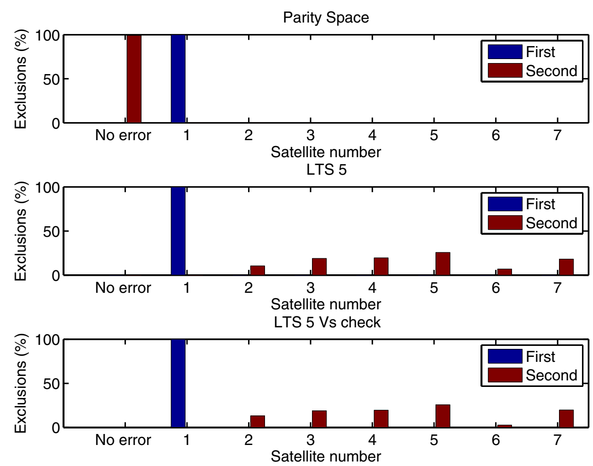


(b) Multipath and impulsive noise errors

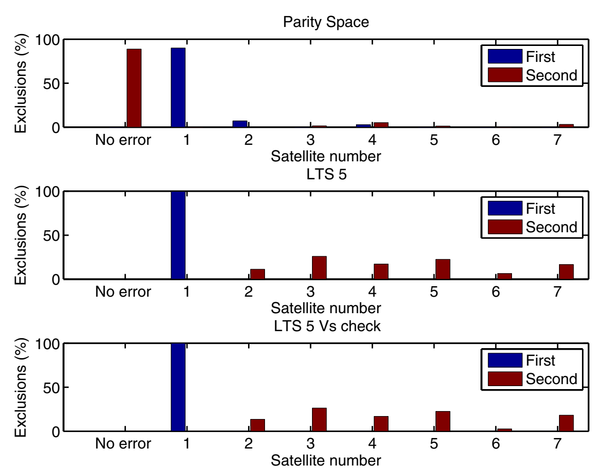
Fig. 3. SPATIAL ERROR DISTRIBUTIONS SHOWING PERCENTAGE OF LARGE ERRORS PRODUCED AT EVERY TEST POINT.



(a) Ramp error



(b) Multipath error



(c) Multipath and impulsive noise errors

Fig. 4. RANGE EXCLUSION PERFORMANCE OF PRESENTED ALGORITHMS

fails to detect these errors, being unable to correctly detect the erroneous satellite measurements when both failures occur simultaneously.

D. Computation Time

Robust methodologies have the drawback of being more time consuming than non robust. Table II shows the computation time of the algorithms under study for the error situations presented. Both robust algorithms last approximately the same. It is observed a general tendency of increasing computation time when the error condition is worsened. Robust algorithms computation times are from 2.7 to 8.4 times the corresponding parity space algorithms.

The maximum update rate of the 3D-LOCUS system is 10 times per second. Since these simulation are accomplished over 2200 measurements, It would not be any decrement on

TABLE II.
COMPUTATION TIME (IN SECONDS) OF THE ALGORITHMS STUDIED UNDER THE FOUR ERROR SITUATIONS CONSIDERED

Error situation	Parity Space	LTS 5	LTS 5 Vs Check
Gaussian noise	9.8958	39.5217	39.8301
Ramp error	15.6884	42.3197	42.3714
Multipath error	16.8589	140.9051	141.2916
Multipath & outliers	18.5713	142.3641	142.4994

this update rate when adding the robust algorithms. It has to be taken into account that the simulated algorithms are not completely optimized, being easy to reduce these computations time. For instance, the maximum number of iterations of the basic algorithm is fixed to 50; this value can be reduced since the usual number of iteration when the algorithm converges to a good result is usually below 10.

VI. DISCUSSION AND CONCLUSION

It has been presented an evaluation of the feasibility of using the traditional parity space method used in GPS against robust methodologies for estimating the user position in a typical configuration of an ultrasound location system corrupted with common errors. It has been shown how the parity space method outperforms robust techniques when considering one single error; whereas robust techniques work better with multiple errors. The proposed velocity of sound monitoring applied to the LTS algorithm seems to be the best choice among tested methods.

Although only one case of failing measurements has been taken into account for each error, the results obtained changing the failing node would be very similar. This can be expected due to the symmetry of the fixed nodes arrangement.

The high percentage of large errors detected in the robust LTS algorithm with small measurement errors seems to indicate that the basic linearized algorithm proposed can be inadequate for applying this technique, or the satellite distribution is not appropriate. It could be studied the possibility of developing a different algorithm whose residuals show a better relationship with the positioning error. Also it could be studied the possibility of applying this technique to a modified residual vector (using the faulty vector, for instance).

Fixed nodes are distributed following a typical triangular lattice. This is very usual in these systems since, if the velocity of sound is not estimated, no error is present in the measurements (apart from gaussian noise), and there is not redundancy (only three nodes), the best results are obtained with a triangular arrangement forming an equilateral triangle with the fixed nodes. This result is easily obtained from the concept of Dilution Of Precision (DOP) [19] which relates measuring and positioning errors. It would be interesting to find whether this arrangement is appropriate when considering sound velocity estimation and/or redundancy, as well as the effect of eliminating one of such measurements.

The analysis of the computation time presented shows that the robust algorithm can be implemented into the 3D-LOCUS system without influencing its update rate.

This paper has shown a comparison between the parity space method and robust methods, detecting pros and cons of the presented algorithms. Its aim is to open the discussion about the method, arrangements, and algorithms that should be employed in ultrasound location systems. It is necessary to extend the research on these areas to enhance the robustness for a real proliferation of these precise, inexpensive, and modular local positioning systems.

ACKNOWLEDGMENT

This work has been funded by the Spanish Ministries of Science, Education and Innovation under PARMEI (Ref. DIP2003-08715-C02-02) and RESELAI (MEC.TIN2006-14896-C02-02) projects.

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