Dynamic Response Estimation of Multilayer Ferroelectret-based Transducers using Lumped-Element Electromechanical Models

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Abstract—In this work, we show that in spite of the anisotropy of the ferroelectret Emfit film and the inhomogeneity of its vibratory behavior, the average frequency response of multilayer transducers in air is fairly well predicted with lumped element models. Transducers prototypes of up to three layers were built using plastic adhesive tapes and characterized by means of interferometric measurements on the uppermost layer surface. Each Emfit layers have been modeled by means of a damped spring-mass system. A good agreement between theory and experimentation was obtained following a constrained parameter identification procedure.

I. INTRODUCTION

The use of multi-layer electrets in transducer design has already been propose since the sensitivity of the device can be enhanced by stacking films [1]. Multilayer sensors and sound receivers have been prepared and characterized using several cellular PP films [2] [3]. Even though single layer devices has been extensively studied, little attention has been paid to the multilayer transducer performance at ultrasonic frequencies.

Ferroelectrets exhibit remarkable characteristics which make them suitable for ultrasonic air applications, where the frequency response and, specifically, the thickness-extension resonance of the device, are essential. A tunable resonant frequency results advantageous for narrow band devices as well as for transducer operated far from resonance. Therefore, there is a need for models which allow us to efficiently estimate the response of ferroelelectret-based transducers of multilayer configuration. Quasi-static coupled models for predicting $d_{33}$ and compliance of cellular ferroelectret films have been reported by several researchers. The most common approach is an extension of the electret microphone model proposed in [4]. It assumes the material composed of alternating air and solid layers with a finite charge density at the boundary [5] [6]. Also, the dynamic response measured using dielectric spectroscopy has been fitted by employing a “black box” view of the piezoelectric resonators [7]. More complex approaches includes static coupled models using finite elements schemes for calculating the deformation of 2 and three-layer electret systems [8] [9] and micromechanical models to predict the macroscopically observed piezoelectric material behavior [10].

In this work, we show that in spite of the anisotropy of the ferroelectret Emfit film and the measured inhomogeneity of its vibratory behavior, the ultrasonic frequency response of multilayer transducers in air, including their multiple resonant frequencies, can be fairly well predicted using lumped-element coupled electromechanical models. Transducers prototypes of up to three layers were built using plastic adhesive tapes and characterized by means of interferometric measurements on the uppermost layer surface. Each Emfit layer of the prototypes has been modeled by means of a damped spring-mass system. A good agreement between theory and experimentation was obtained following a constrained parameter identification procedure that takes into account the significant effect of the adhesive tapes mass on the transducer response.

II. MATERIALS AND METHODS

A. Transducer prototypes

The ferroelectret material used in this work has been provided by Emfit Ltd. It is detailed as Emfit Film, product number HS-03-20BR AL1, metalized on one side only and without a preaging process. The film is fixed using plastic adhesive tapes to a solid substrate. This way, the fabrication process has proved to be fast, repeatable and reliable.

In this work, we have fabricated and characterized three different flat square transducers, i.e. transducers of one, two and three layers. In the construction process of the prototypes we have employed two types of plastic adhesive tapes, namely, a through-thickness conductive tape (ZCPT, 8.915 mg/cm²) and a XYZ-axis electrically conductive plastic tape (ECPT,11.874 mg/cm²). The thickness of the tapes are ≈150 µm and ≈50 µm respectively. Figure 1 shows a sketch of the fabricated multilayer transducers and the different parts they consist of. Notice that the Emfit films are positioned with their non-metalized side in contact and electrically connected following an antiparallel configuration. Therefore, the piezoelectric stresses acting on the electrodes of all films are in phase. In this manner, the individual thickness change of each layer will add up and, consequently, the acoustic output is maximized.

In this work, we show that in spite of the anisotropy of the ferroelectret Emfit film and the measured inhomogeneity of its vibratory behavior, the ultrasonic frequency response of multilayer transducers in air, including their multiple resonant frequencies, can be fairly well predicted using lumped-element coupled electromechanical models. Transducers prototypes of up to three layers were built using plastic adhesive tapes and characterized by means of interferometric measurements on the uppermost layer surface. Each Emfit layer of the prototypes has been modeled by means of a damped spring-mass system. A good agreement between theory and experimentation was obtained following a constrained parameter identification procedure that takes into account the significant effect of the adhesive tapes mass on the transducer response.
**B. Experimental Setup**

Figure 2 illustrates a block diagram of the experimentation equipment used in this work. This instrumentation consists of 4 main parts, i.e. the transducer excitation equipment, the acoustic pressure measurement channel, the laser-based Doppler Vibrometer and the servo-controlled motion units. In order to acquire the frequency response of the prototypes, interferometric measurements were carried on the upper layer surface. Wideband linear chirp signals from 30 kHz to 400 kHz were applied.

**III. DYNAMIC ACTUATOR RESPONSE**

In spite of the fact that a better understanding of the physical behavior of the ferroelectret film is provided by already proposed models, not enough end-user practical considerations, based on experimentation, are given with regard to the equivalent dynamic mass of the film, the dynamic stiffness, the resonant frequency tuning and the effect of the glue used in the fabrication process. Besides, little attention has been paid to the statistical nature of the ferroelectret films, which results in an increased difficulty when model parameters identification is intended.

In this section, we develop an electromechanical coupled model of lumped parameters which allow the users of ferroelectret films to customize a multilayer transducer and predict its dynamic response versatility.

**A. Multiple DoF Electromechanical Model**

The electromechanical model of the Emfit film describes the displacement of the upper surface of the film \((x_1)\) when an ac external voltage is applied to the transducer’s electrodes. Figure 3 (right) shows a cross section of a 1 degree-of-freedom (DoF) transducer prototype. The central part of the film, which is made of air cavities, is modeled as a viscoelastic material by means of a spring in parallel with a linear damper. The upper layer of vaporized aluminum and the adhesive tapes act as electrodes.

\[
m_1 \text{ is the effective dynamic mass of the transducer. It is considered as the sum of the upper electrode mass and a fraction of the foamy central part.}
\]

\[
k_1 \text{ is the combined dynamic stiffness due to the Emfit’s air voids and its solid part.}
\]

\[
C_1 \text{ is the damping coefficient which encloses the viscous losses in the internal structure}
\]

\[
C_{\text{air}} \text{ includes the mechanical impedance of the air.}
\]

\[
V \text{ is the driving voltage applied to the transducer.}
\]

\[
x_0 \text{ is the thickness of the polarized Emfit material at equilibrium, as provided by the manufacturer. A typical value of } \sim 70 \mu \text{m is given.}
\]

As the ferroelectret film is permanently polarized, it is initially preloaded by an electrostatic force \(F_0\) which is given by \(F_0 = k_1 \delta = \frac{q_0^2}{2\varepsilon A}\), where \(q_0\), \(\varepsilon\) and \(A\), are the stored charge at equilibrium, the dielectric permittivity and the transducer area, respectively. \(\delta\) is the respective initial thickness variation because of the electrostatic attraction force \(F_0\). At equilibrium, no spring response from either the adhesive or the film is considered. In addition to this, the resultant dynamic system model has been obtained under the assumption that the vibration takes place around \(x_0\). The capacitance of the ferroelectret film is considered to vary according to: \(\frac{\varepsilon A}{x_0} \). Using a charge formulation and the generalized coordinates \(x_1\) and \(q_1\), the Lagrange’s Equations read:

\[
m \ddot{x} + (C_1 + C_{\text{air}}) \dot{x} + k_1 (x + \delta) - \frac{(q_0 + q)^2}{2\varepsilon A} = F \tag{1}
\]

\[
x_0 - x = \frac{(q_0 + q)}{\varepsilon A} \tag{2}
\]

In the equations, note the quadratic relationship between the force and voltage. It is also shown that the motion of the mass tends to complicate this relationship. Assuming that \(q_1 \ll q_0\), \(x_1 \ll x_0\) and that the driving voltage may be divided into a large bias voltage \(V_{dc}\) and a signal voltage \(V_{ac}\), and that the
external forces $F$ are null in the actuator mode of operation, it is possible to linearize the equations (1) and (2) so that:

$$m\ddot{x}_1 + (C_1 + C_{air})\dot{x}_1 + (k_1 - T)x = \frac{q_0}{x_0}V_{ac}$$  \hspace{1cm} (3)

where $T = \frac{q_0^2}{\epsilon A x_0}$. This resultant equation approximates the dynamical response due to an external ac excitation voltage. Note that the stiffness of the systems depends on the initial stored charge $q_0$.

Figure 4 shows the schematic design of a three-layer transducers in antiparallel configuration. Following the same procedure described for the single DoF model under similar assumptions, the resultant $n \times n$ matrix equation of motion of an $n$-layer transducer is:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = x = \{x_1, x_2, ..., x_n\}^T, [M] = \text{diag}[m_1, m_2, ..., m_n]$$

and the damping and stiffness matrices are:

$$[C] = \begin{bmatrix}
C_1 + C_{air} & -C_1 & \cdots & 0 \\
-C_1 & C_1 + C_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & -C_{n-1} \\
0 & 0 & \cdots & -C_n - 1 + C_n
\end{bmatrix}$$

$$[K] = \begin{bmatrix}
k_1 - T & -k_1 + T & \cdots & 0 \\
-k_1 & k_1 + k_2 - T & \cdots & 0 \\
\vdots & \vdots & \ddots & -k_{n-1} + T \\
0 & 0 & \cdots & -k_{n-1} + k_n - T
\end{bmatrix}$$

Notice that the same voltage is applied to each layer.

IV. RESULTS

A. Transducers Characterization

The dynamic response of 256 points on the upper surface layer of the transducers was measured by interferometry. Figure 5 shows the average response obtained for each transducer fabricated. Notice that the number of layers corresponds to the number of resonances. Besides, the sensitivity at frequencies between 30 and 100 kHz is enhanced for both two- and three-layer transducers. Acoustic measurements (not shown in this work) of the directivity pattern at their respective first resonances well agree with that of a piston-like vibratory pattern. However, the variability of the multilayer transducers response after the first resonance becomes higher than that of a single foil transducers near resonance.

B. Parameter Identification Procedure

The single layer model was first identified and the parameters obtained, i.e. spring constant, damping ratio and dynamic mass, were utilized for the identification of the two- and three-layer models. The first step in the parameter identification process was determining the effective dynamic mass $m_3$ of a single-layer transducer. A small mass of adhesive tape of approximately $\Delta m = 14.3$ mg of weight was stuck on the upper electrode of the transducer prototype. As a consequence, the resonant frequency was shifted to 165 kHz. Using the relationship expressed in equation (7), which relates the damped resonant frequency shift to the added mass, it was possible to determine an effective dynamic mass $m_1$ of $m = 5.49$ mg. This value is about the 40% of the total mass of the Emfit film used in the emitter fabrication, calculated from the density and thickness given by the manufacturer.

$$\frac{m_1 + \Delta m}{m_1} = \left(\frac{\omega_r m_1}{\omega_r m_1 + \Delta m}\right)^2$$

Subsequently, by means of a least squares fitting technique, the rest of unknown parameters of the 1-DoF model were found, i.e. $q_0$, $C = C_1 + C_{air}$ and $k_3$ as illustrated in equation (3). This procedure was carried out by comparing the frequency response of each measured point on the transducer with the model output. Figure 7 shows the statistical parameters distribution of the identified parameters. Apart from the damping coefficient, the rest of parameters exhibit a fairly normal distribution. Besides, a value of $q_0 = 1.087 \times 10^{-7}$ C corresponds to an internal stored voltage of $V_0 = q_0/C_0 = 1235$, 7 volts. $C_0$ is the capacitance of the transducers, estimated from the information given by the manufacturer (22 pF/cm²). In the frequency domain, a residual error not greater than 15% between the real data and the estimated transfer function was obtained in the 30–350 kHz frequency range (see figure 6, top picture).
Parameters $m_1, k_1$ and $q_0$ were constrained during the identification of the 2-DoF and 3DoF models, according to their statistical distribution. Besides, the mass of the adhesive tapes were included in the models according to:

$$m_2 = m_{ECPT} + 2(m_1 - m_{Al}) \approx 57 \text{mg} \quad \text{and} \quad m_3 = m_{ZCPT} + 2m_1 \approx 46.46 \text{mg},$$

where $m_{Al}$ is the mass of the aluminum electrode, which is calculated to be $\approx 0.216 \text{ mg}$. This assumption implies that not only the adhesive tape masses modify the dynamic response, but also some part of the polypropylene of the Emfit layers attached to them. Table I summarizes the whole set of identified parameters. Figure 6 shows the fitting between measurements and models output.

V. CONCLUSIONS

Lumped-element electromechanical models provide valuable information about the location of the resonant frequencies of multilayer transducers. In spite of the statistical nature of the ferroelectret film, a fairly good agreement with the measured response of multilayer prototypes has been achieved without significant changes neither in the dynamic masses nor in the spring constants. The obtained damping coefficients seems to indicate that a higher amount of energy is dissipated in the lower layers. Future research is required in order to properly explain the lack of fitting in the amplitude around the first resonance. This could be attributable to different causes, such as nonlinear effects not included in the models, the influence of the resistivity of the adhesive tapes at low frequencies, among others. Research is being conducted in order to validate the models when different voltages are applied at each layer.

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REFERENCES